

Computation with Absolutely No Space Overhead

Lane Hemaspaandra¹ Proshanto Mukherji¹ Till Tantau²

¹Department of Computer Science
University of Rochester

²Fakultät für Elektrotechnik und Informatik
Technical University of Berlin

Developments in Language Theory Conference, 2003



Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful



Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful



Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful



Outline

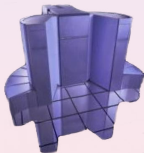
- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful



The Standard Model of Linear Space

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
|---|---|---|---|---|---|---|---|



Turing machine

Characteristics

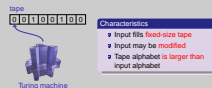
- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

Computation with Absolutely No Space Overhead

└ The Model of Overhead-Free Computation

└└ The Standard Model of Linear Space

└└└ The Standard Model of Linear Space



1. Point out that \$ is a marker symbol.
2. Stress the larger tape alphabet.

The Standard Model of Linear Space

tape

| | | | | | | | |
|----|---|---|---|---|---|---|---|
| \$ | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
|----|---|---|---|---|---|---|---|



Turing machine

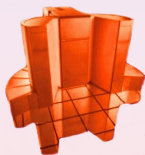
Characteristics

- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape

| | | | | | | | |
|----|---|---|---|---|---|---|---|
| \$ | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
|----|---|---|---|---|---|---|---|



Turing machine

Characteristics

- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape

| | | | | | | | |
|----|---|---|---|---|---|---|----|
| \$ | 0 | 1 | 0 | 0 | 1 | 0 | \$ |
|----|---|---|---|---|---|---|----|



Turing machine

Characteristics

- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape

| | | | | | | | |
|----|---|---|---|---|---|---|----|
| \$ | 0 | 1 | 0 | 0 | 1 | 0 | \$ |
|----|---|---|---|---|---|---|----|



Turing machine

Characteristics

- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape

| | | | | | | | |
|----|----|---|---|---|---|---|----|
| \$ | \$ | 1 | 0 | 0 | 1 | 0 | \$ |
|----|----|---|---|---|---|---|----|



Turing machine

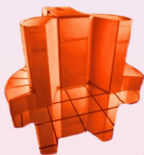
Characteristics

- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape

| | | | | | | | |
|----|----|---|---|---|---|---|----|
| \$ | \$ | 1 | 0 | 0 | 1 | 0 | \$ |
|----|----|---|---|---|---|---|----|



Turing machine

Characteristics

- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape

| | | | | | | | |
|----|----|---|---|---|---|----|----|
| \$ | \$ | 1 | 0 | 0 | 1 | \$ | \$ |
|----|----|---|---|---|---|----|----|



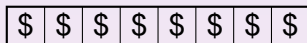
Turing machine

Characteristics

- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape



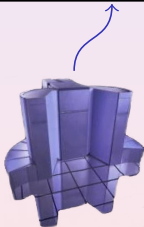
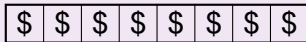
Turing machine

Characteristics

- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

The Standard Model of Linear Space

tape

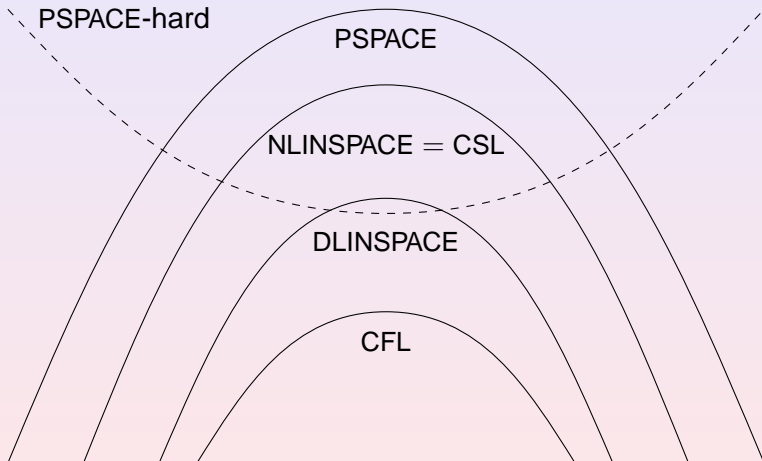


Turing machine

Characteristics

- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

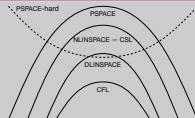
Linear Space is a Powerful Model



Computation with Absolutely No Space Overhead

- └ The Model of Overhead-Free Computation
 - └ The Standard Model of Linear Space
 - └ Linear Space is a Powerful Model

Linear Space is a Powerful Model



1. Explain CSL.
2. Point out the connections to formal language theory.

Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful



Our Model of “Absolutely No Space Overhead”

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
|---|---|---|---|---|---|---|---|



Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **equals** input alphabet

Our Model of “Absolutely No Space Overhead”

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
|---|---|---|---|---|---|---|---|



Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **equals** input alphabet

Our Model of “Absolutely No Space Overhead”

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
|---|---|---|---|---|---|---|---|



Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **equals** input alphabet

Our Model of “Absolutely No Space Overhead”

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
|---|---|---|---|---|---|---|---|



Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **equals** input alphabet

Our Model of “Absolutely No Space Overhead”

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
|---|---|---|---|---|---|---|---|



Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **equals** input alphabet

Our Model of “Absolutely No Space Overhead”

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
|---|---|---|---|---|---|---|---|



Turing machine

Characteristics

- Input fills **fixed-size tape**
- Input may be **modified**
- Tape alphabet **equals** input alphabet

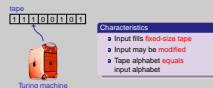
Computation with Absolutely No Space Overhead

└ The Model of Overhead-Free Computation

└└ Our Model of Absolutely No Space Overhead

└└└ Our Model of “Absolutely No Space Overhead”

Our Model of “Absolutely No Space Overhead”



1. Point out that no markers are used.

Our Model of “Absolutely No Space Overhead”



Turing machine

Intuition

- Tape is used like a RAM module.

Definition of Overhead-Free Computations

Definition

A Turing machine is **overhead-free** if

- 1 it has only a single tape,
- 2 writes only on input cells,
- 3 writes only symbols drawn from the input alphabet.



Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in

DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ ,

DOF_{poly} if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.

NOF is the nondeterministic version of DOF ,

NOF_{poly} is the nondeterministic version of DOF_{poly} .



Computation with Absolutely No Space Overhead

└ The Model of Overhead-Free Computation

└ Our Model of Absolutely No Space Overhead

└ Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in**DOF** if L is accepted by a deterministic overhead-free machine with input alphabet Σ .**DOF_{poly}** if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.**NDOF** is the nondeterministic version of DOF.

1. Joke about German pronunciation

Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in

DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ ,

DOF_{poly} if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.

NOF is the nondeterministic version of DOF,

NOF_{poly} is the nondeterministic version of DOF_{poly}.



Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in

DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ ,

DOF_{poly} if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.

NOF is the nondeterministic version of DOF,

NOF_{poly} is the nondeterministic version of DOF_{poly}.



Computation with Absolutely No Space Overhead

└ The Model of Overhead-Free Computation

└ Our Model of Absolutely No Space Overhead

└ Overhead-Free Computation Complexity Classes

Definition
A language $L \subseteq \Sigma^*$ is in

- DOF** if L is accepted by a deterministic overhead-free machine with input alphabet Σ .
- DOF_{poly}** if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.
- NOF** is the nondeterministic version of DOF.

DOF_{poly} is the nondeterministic version of DOF_{poly}.

1. Stress meaning of D and N.

Overhead-Free Computation Complexity Classes

Definition

A language $L \subseteq \Sigma^*$ is in

DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ ,

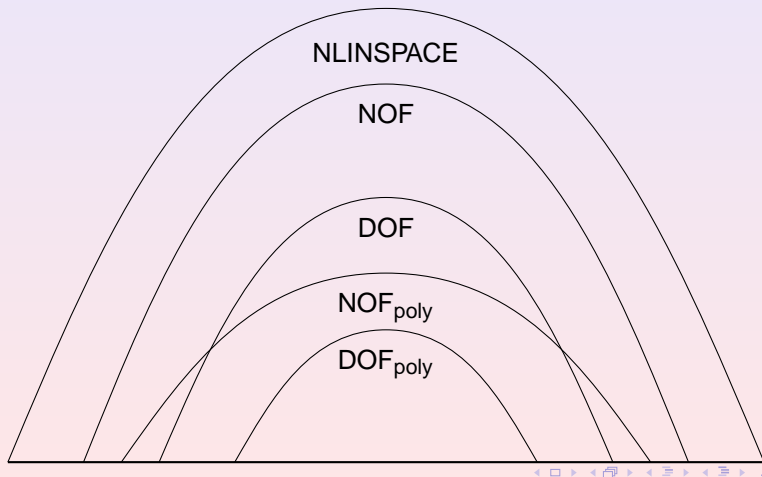
DOF_{poly} if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time.

NOF is the nondeterministic version of DOF,

NOF_{poly} is the nondeterministic version of DOF_{poly}.



Simple Relationships among Overhead-Free Computation Classes



Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful



Palindromes Can be Accepted in an Overhead-Free Way

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
|---|---|---|---|---|---|---|---|



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

Advance left end marker

Find right end marker

Advance right end marker

2004-10-08

Computation with Absolutely No Space Overhead

└ The Power of Overhead-Free Computation

└ Palindromes

└ Palindromes Can be Accepted in an Overhead-Free Way

Palindromes Can be Accepted in an Overhead-Free Way



Algorithm

Phase 1:

Compare first and last bit
Place left end marker
Place right end marker

Phase 2:

Compare bits next to end markers
Find left end marker
Advance left end marker
Find right end marker
Advance right end marker

Use 3 minutes.

Palindromes Can be Accepted in an Overhead-Free Way

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
|---|---|---|---|---|---|---|---|



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
|---|---|---|---|---|---|---|---|



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
|---|---|---|---|---|---|---|---|



overhead-free machine

Algorithm

Phase 1:

- Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:

- Compare bits next to end markers
- Find left end marker
- Advance left end marker
- Find right end marker
- Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
|---|---|---|---|---|---|---|---|



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
|---|---|---|---|---|---|---|---|



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
|---|---|---|---|---|---|---|---|



overhead-free machine

Algorithm

Phase 1:

- Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:

- Compare bits next to end markers
- Find left end marker
- Advance left end marker
- Find right end marker
- Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
|---|---|---|---|---|---|---|---|



overhead-free machine

Algorithm

Phase 1:

- Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:

- Compare bits next to end markers
- Find left end marker
- Advance left end marker
- Find right end marker
- Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
|---|---|---|---|---|---|---|---|



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
|---|---|---|---|---|---|---|---|



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
|---|---|---|---|---|---|---|---|



overhead-free machine

Algorithm

Phase 1:

- Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:

- Compare bits next to end markers
- Find left end marker
- Advance left end marker
- Find right end marker**
- Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
|---|---|---|---|---|---|---|---|



overhead-free machine

Algorithm

Phase 1:

- Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:

- Compare bits next to end markers
- Find left end marker
- Advance left end marker
- Find right end marker
- Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
|---|---|---|---|---|---|---|---|



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
|---|---|---|---|---|---|---|---|



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
|---|---|---|---|---|---|---|---|



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

Advance left end marker

Find right end marker

Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

tape

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|



overhead-free machine

Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

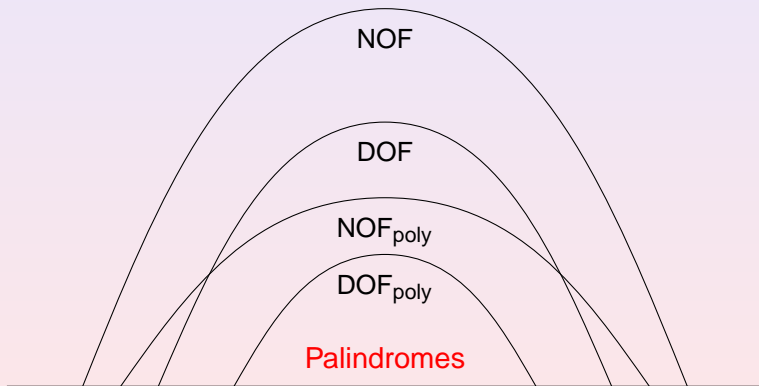
Find left end marker

Advance left end marker

Find right end marker

Advance right end marker

Relationships among Overhead-Free Computation Classes



Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful



A Review of Linear Grammars

Definition

A grammar is **linear** if it is context-free and there is only one nonterminal per right-hand side.

Example

$G_1: S \rightarrow 00S0 \mid 1$ and $G_2: S \rightarrow 0S10 \mid 0$.

Definition

A grammar is **deterministic** if
“there is always only one rule that can be applied.”

Example

$G_1: S \rightarrow 00S0 \mid 1$ is deterministic.

$G_2: S \rightarrow 0S10 \mid 0$ is **not** deterministic.

A Review of Linear Grammars

Definition

A grammar is **linear** if it is context-free and there is only one nonterminal per right-hand side.

Example

$G_1: S \rightarrow 00S0 \mid 1$ and $G_2: S \rightarrow 0S10 \mid 0$.

Definition

A grammar is **deterministic** if
“there is always only one rule that can be applied.”

Example

$G_1: S \rightarrow 00S0 \mid 1$ is deterministic.

$G_2: S \rightarrow 0S10 \mid 0$ is **not** deterministic.

Computation with Absolutely No Space Overhead

└ The Power of Overhead-Free Computation

└ Linear Languages

└ A Review of Linear Grammars

A Review of Linear Grammars

Overview

A grammar is *linear* if it is context-free and there is only one nonterminal per right-hand side.

Examples

$G_1: S \rightarrow 00S0 \mid 1$ and $G_2: S \rightarrow 0S10 \mid 0$.

Definition

A grammar is *deterministic* if "there is always only one rule that can be applied."

Example

$G_1: S \rightarrow 00S0 \mid 1$ is deterministic.
 $G_2: S \rightarrow 0S10 \mid 0$ is *not* deterministic.

Just explain intuition.

Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every deterministic linear language is in DOF_{poly} .



Metalinear Languages Can Be Accepted in an Overhead-Free Way

Definition

A language is **metalinear** if it is the concatenation of linear languages.

Example

TRIPLE-PALINDROME = $\{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

Theorem

Every metalinear language is in NOF_{poly} .



Metalinear Languages Can Be Accepted in an Overhead-Free Way

Definition

A language is **metalinear** if it is the concatenation of linear languages.

Example

TRIPLE-PALINDROME = $\{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

Theorem

Every metalinear language is in NOF_{poly} .



Metalinear Languages Can Be Accepted in an Overhead-Free Way

Definition

A language is **metalinear** if it is the concatenation of linear languages.

Example

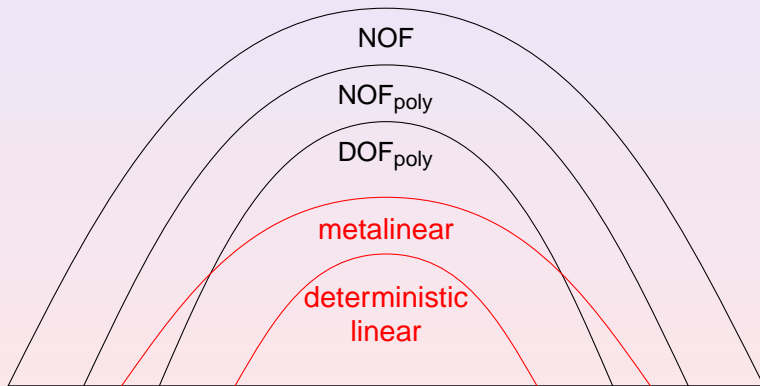
TRIPLE-PALINDROME = $\{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

Theorem

Every metalinear language is in NOF_{poly} .



Relationships among Overhead-Free Computation Classes

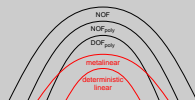


2004-10-08

Computation with Absolutely No Space Overhead

- └ The Power of Overhead-Free Computation
 - └ Linear Languages
 - └ Relationships among Overhead-Free Computation Classes

Relationships among Overhead-Free Computation Classes



1. Skip next subsection if more than 18 minutes have passed.

Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - **Context-Free Languages with a Forbidden Subword**
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful



Definition of Almost-Overhead-Free Computations

Definition

A Turing machine is **almost-overhead-free** if

- 1 it has only a single tape,
- 2 writes only on input cells,
- 3 writes only symbols drawn from the input alphabet plus one special symbol.



Definition of Almost-Overhead-Free Computations

Definition

A Turing machine is **almost-overhead-free** if

- 1 it has only a single tape,
- 2 **writes only on input cells,**
- 3 writes only symbols drawn from the input alphabet plus one special symbol.



Definition of Almost-Overhead-Free Computations

Definition

A Turing machine is **almost-overhead-free** if

- 1 it has only a single tape,
- 2 writes only on input cells,
- 3 writes only symbols drawn from the input alphabet plus one special symbol.



Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

Theorem

*Let L be a context-free language with a forbidden word.
Then $L \in \text{NOF}_{\text{poly}}$.*

» Skip proof



Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

Theorem

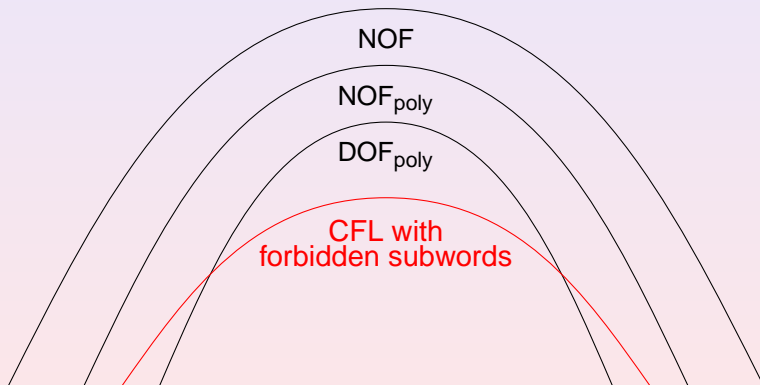
*Let L be a context-free language with a forbidden word.
Then $L \in \text{NOF}_{\text{poly}}$.*

Proof.

Every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time. □



Relationships among Overhead-Free Computation Classes



Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
 - Linear Space is Strictly More Powerful



Overhead-Free Languages can be PSPACE-Complete

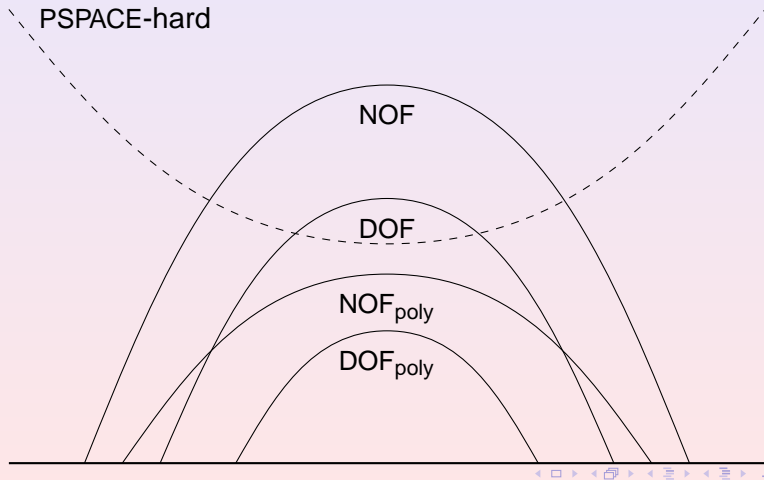
Theorem

DOF *contains languages that are complete for PSPACE.*

► Proof details



Relationships among Overhead-Free Computation Classes



Outline

- 1 The Model of Overhead-Free Computation
 - The Standard Model of Linear Space
 - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
 - Palindromes
 - Linear Languages
 - Context-Free Languages with a Forbidden Subword
 - Languages Complete for Polynomial Space
- 3 **Limitations of Overhead-Free Computation**
 - **Linear Space is Strictly More Powerful**



Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

Theorem

$\text{DOF} \subsetneq \text{DLINSPACE}$.

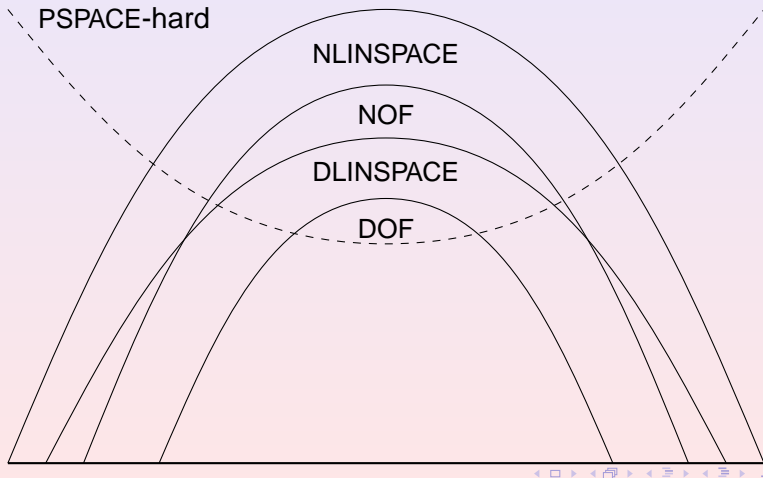
Theorem

$\text{NOF} \subsetneq \text{NLINSPACE}$.

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.



Relationships among Overhead-Free Computation Classes



Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

Conjecture

DOUBLE-PALINDROMES \notin DOF.

Conjecture

$\{ww \mid w \in \{0,1\}^*\} \notin$ NOF.

Proving the first conjecture would show $\text{DOF} \subsetneq \text{NOF}$.



Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

Theorem

$\text{DOUBLE-PALINDROMES} \in \text{DOF}.$

Conjecture

$\{ww \mid w \in \{0,1\}^*\} \notin \text{NOF}.$

Proving the first conjecture would show $\text{DOF} \subsetneq \text{NOF}.$



Summary

- Overhead-free computation is a more faithful **model of fixed-size memory**.
- Overhead-free computation is **less powerful** than linear space.
- **Many** context-free languages can be accepted by overhead-free machines.
- We conjecture that **all** context-free languages are in NOF_{poly} .
- Our results can be seen as new results on the power of **linear bounded automata with fixed alphabet size**.



Computation with Absolutely No Space Overhead

Summary

Summary

Summary

Summary

- Overhead-free computation is a more faithful model of fixed-size memory.
- Overhead-free computation is *less powerful* than linear space.
- Many* context-free languages can be accepted by overhead-free machines.
- We conjecture that *all* context-free languages are in NOF_{poly} .
- Our results can be seen as new results on the power of *linear bounded automata with fixed alphabet size*.

1. Point out result concerning all context-free languages.
2. Relationship to restart automata.

For Further Reading



A. Salomaa.

Formal Languages.

Academic Press, 1973.



E. Dijkstra.

Smoothsort, an alternative for sorting in situ.

Science of Computer Programming, 1(3):223–233, 1982.



E. Feldman and J. Owings, Jr.

A class of universal linear bounded automata.

Information Sciences, 6:187–190, 1973.



P. Jančar, F. Mráz, M. Plátek, and J. Vogel.

Restarting automata.

FCT Conference 1995, LNCS 985, pages 282–292. 1995.



For Further Reading



A. Salomaa.

Formal Languages.

Academic Press, 1973.



E. Dijkstra.

Smoothsort, an alternative for sorting in situ.

Science of Computer Programming, 1(3):223–233, 1982.



E. Feldman and J. Owings, Jr.

A class of universal linear bounded automata.

Information Sciences, 6:187–190, 1973.



P. Jančar, F. Mráz, M. Plátek, and J. Vogel.

Restarting automata.

FCT Conference 1995, LNCS 985, pages 282–292. 1995.



For Further Reading



A. Salomaa.

Formal Languages.

Academic Press, 1973.



E. Dijkstra.

Smoothsort, an alternative for sorting in situ.

Science of Computer Programming, 1(3):223–233, 1982.



E. Feldman and J. Owings, Jr.

A class of universal linear bounded automata.

Information Sciences, 6:187–190, 1973.



P. Jančar, F. Mráz, M. Plátek, and J. Vogel.

Restarting automata.

FCT Conference 1995, LNCS 985, pages 282–292. 1995.



For Further Reading



A. Salomaa.

Formal Languages.

Academic Press, 1973.



E. Dijkstra.

Smoothsort, an alternative for sorting in situ.

Science of Computer Programming, 1(3):223–233, 1982.



E. Feldman and J. Owings, Jr.

A class of universal linear bounded automata.

Information Sciences, 6:187–190, 1973.



P. Jančar, F. Mráz, M. Plátek, and J. Vogel.

Restarting automata.

FCT Conference 1995, LNCS 985, pages 282–292. 1995.



Appendix Outline

4

Appendix

- Complete Languages
- Improvements for Context-Free Languages
- Abbreviations



Overhead-Free Languages can be PSPACE-Complete

Theorem

DOF contains languages that are complete for PSPACE.

Proof.

- 1 Let $A \in \text{DLINSPACE}$ be PSPACE-complete.
Such languages are known to exist.
- 2 Let M be a linear space machine that accepts $A \subseteq \{0, 1\}^*$
with tape alphabet Γ .
- 3 Let $h: \Gamma \rightarrow \{0, 1\}^*$ be an isometric, injective
homomorphism.
- 4 Then $h(L)$ is in DOF and it is PSPACE-complete. □

Improvements

Theorem

- 1 $\text{DCFL} \subseteq \text{DOF}_{\text{poly}}$.
- 2 $\text{CFL} \subseteq \text{NOF}_{\text{poly}}$.



Explanation of Different Abbreviations

| | |
|---------------------|--|
| DOF | Deterministic Overhead-Free. |
| NOF | Nondeterministic Overhead-Free. |
| DOF _{poly} | Deterministic Overhead-Free, polynomial time. |
| DOF _{poly} | Nondeterministic Overhead-Free, polynomial time. |

Table: Explanation of what different abbreviations mean.

